

Overview

This report serves members of the Lupascu Group continuing the research on entangling gates between transmon qubits. The experiment discussed herein aims to apply an iSWAP operation to two neighboring transmons through a parametric coupling scheme. Design choices and noise contributions are emphasized, while system control and readout are abstracted to future work.

Literature

Work in the transmon regime has been well documented since [J. Koch et al. \(2007\)](#), which outlines the cQED of the charge qubit in the transmon regime, and includes a synopsis of control, readout, and noise analysis techniques. Modern work is closely related to the project at hand, including theory by [Yan et al \(2018\)](#) in which the dynamics of a tunable coupler are defined, and the experiment by [Sung et al. \(2021\)](#) in which a high-fidelity iSWAP gate is demonstrated between two qubits via a tunable coupler. Our experiment utilizes the shared circuit in these two papers as the foundation for an experimental iSWAP operation between qutrits.

Methodology

Before designing the transmon devices, a Hamiltonian equivalence needs to be established between the circuits discussed in prior work and that of potential design choices. Namely, grounded and floating transmons which are both used frequently are canonically thought of as having identical behaviors, although a more rigorous proof helps determine which may be of greater interest for this experiment. See [Proving Equivalence Between Transmon Hamiltonian](#) for more information on mapping grounded and floating transmon circuits.

Once Hamiltonian equivalence is established, a design choice is made to emulate the successful experiment done by Sung et. al. This design consists of three adjacent transmons, where the coupling of the two exterior devices is determined by tuning the frequency of the middle device. These transmons embody the popular X-mon design. The coupler and right transmon are flux-tunable via applying an external magnetic flux through the SQUID. The $0 \rightarrow 1$ frequency goes as $\omega = \omega_{max} \sqrt{\cos(\pi\Phi_{ext}/\Phi_0)}$. The orientation of the control and readout lines are determined by the wiring of the Lupascu group's dilution refrigerator (DR). One resulting difference from the Sung et. al paper is that the coupler readout resonator is coupled to the **bottom** of the coupler, instead of one of the teeth. The design is polished in the Simulation phase below.

Noise is among our primary concerns during a multi-device gate experiment where fidelity is already low. In particular, the noise contributed by the external flux in the tunable coupler's SQUID is highly important due to the use of the flux bias line during parametric driving to quickly tune the effective coupling between the target and control qutrits. Among other decoherence channels, flux noise is extensively addressed in [Transmon Decoherence](#). Generally, for a noise contribution λ , the relaxation time is given by $T_1^{-1}(\lambda)_{i,j} = \Gamma = \frac{1}{\hbar^2} |\langle i|A|j \rangle|^2 P(\omega)_\lambda$ where A is the coupling energy between states i and j , and $P(\omega)$ is the noise source power spectrum density. Below are the various noise contributions which are known to dominate our device errors channels.

Noise Source	$\Gamma_{i \rightarrow j}$
Spontaneous Emission	$\Gamma = \frac{d^2 \omega_{ij}^3}{12\pi \epsilon_0 \hbar c^3}; \quad d = 2eL$
Purcell Effect	$\Gamma = \frac{\gamma_{ij}}{2} + iS(\omega); \quad \gamma_{\kappa}^{(i,i+1)} = \kappa \frac{g_{i,i+1}^2}{(\omega_{i,i+1} - \omega_r)^2}$
Capacitive Coupling	$S_V(\omega) = \hbar \omega \text{Re}[Z(\omega)] (1 + \coth \frac{\hbar \omega}{k_B T})$
Flux Coupling	$S_{\Phi_n}(\omega) = M^2 \frac{\hbar \omega}{\omega^2 L^2} \text{Re}[Z_t(\omega)] \left[\coth \frac{\hbar \omega}{2k_B T} + 1 \right]$

Further, diabatic noise contributes to pure dephasing. In general, the dephasing time due to a noise contribution λ is given by $T_2(\lambda)_{ij} = \frac{\hbar}{A_\lambda} \left| \frac{\partial E_{ij}}{\partial \lambda} \right|^{-1}$.

Noise Source	1/f Amplitude	$T_2^{i \rightarrow j}$
Charge	$A_{n_g} = 2.9 \times 10^{-4}$	$T_2^{i \rightarrow j} \approx \frac{4\hbar}{e^2 \pi \epsilon_j }$
Flux	$A_\Phi = 10^{-6}$	$T_2^{i \rightarrow j} \approx \frac{\hbar \Phi_0^2}{A_\Phi^2 \pi^4 \sqrt{2E_{J\Sigma} E_C} d^2 - 1 }$
Critical Current	$A_{I_c} = 10^{-7}$	$T_2^{i \rightarrow j} \approx \frac{\hbar}{A_{I_c}} \left \sqrt{E_c \hbar / e I_c} \right ^{-1}$

Purcell filters were considered, but calculations of noise magnitudes in our systems, in addition to the physical simplicity of our device, allow us to bypass them.

Simulations

Optimized Gate

Using [Qutip](#), the gate is simulated and optimized via trial and error. This gives parameters for E_C , E_J , and the couplings between the transmons. The simulation details can be found on the document titled [Optimized Gate Simulation](#).

Capacitance

Simulating capacitance allows us to design the geometry of the device in such a way as to maximize the desired couplings and minimize the undesired couplings. The capacitance simulation was executed using the Qiskit Metal wrapper of the Ansys Q3D software. Details of this process are found in the document titled [Capacitances from Optimized Gate Simulation](#).

Decoherence

Noise is simulated in two places. Relaxation times are simulated using the convenient Python package [scqubits](#), as described in the document [Calculating T1 Times](#). The other simulation, regarding Dephasing times, uses Mathematica, and is explained in [Calculating T2 Times](#).

Parameters

The resulting parameters, collected via optimizing the above simulations, may be found on the [Device Summary](#).

Next Steps

The work thus far comprises the first steps towards a successful experiment. However, there is more work to be done on simulating the entire device behavior. In particular, simulations of the effect of flux noise (contributed by tuning the coupler SQUID) on the system Hamiltonian have yet to be completed, and are very important for anticipating reliable decoherence times and gate speeds. Further, there are cosmetic issues regarding the CAD layout for the device. The lines visible on the simulation topic documents are not finalized, and should be tuned before fabrication. Finally, characterization devices, such as standalone transmons should be designed and included in fabrication, as well as similar devices with two fixed frequency transmons.